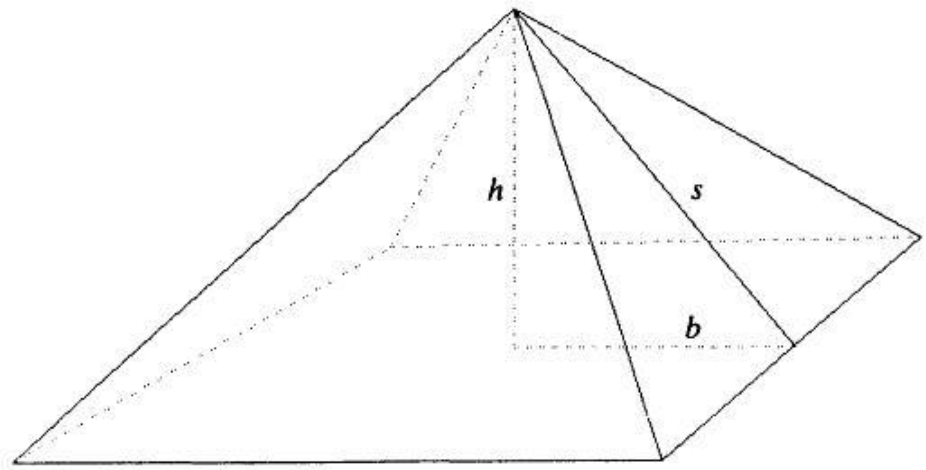

Figure 1 | A square pyramid, image from Markowski 7

Geometry tells us that if the area of a square of length equal to the height of the pyramid is equal to the area of a face of the pyramid, then we have

$$h^2 = sb$$

where h^2 is the area of the square and sb is the area of the triangle, both by definition. By the Pythagorean Theorem and inspection of the above figure we see that

$$h^2 + b^2 = s^2.$$

Now let $r = s/b$. Then we can show that

$$\left(\frac{h}{b}\right)^2 = r \text{ and } \left(\frac{h}{b}\right)^2 + 1 = r^2.$$

Rearranging this, we get

$$r^2 - r - 1 = 0,$$

the only solution to which is $\Phi = \frac{1+\sqrt{5}}{2}$, as demonstrated here.

Proof adapted from Markowski 7.